*"Have no fear of perfection—you will never reach it."*

S. Dali

# 4

# The hydrodynamics of marbling art

## 4.1 Introduction

Marbling is an ancient art form that has evolved across diverse cultures [259–261]. It has traditionally been used to decorate paper for bookbinding or authenticating important documents. Two famous styles of marbling are *ebru*, which uses thickened viscous water, and *suminagashi*, which uses unthickened plain water [262]. A key distinction between the two styles is that ebru-style marbling is more structured with vibrant colors, while suminagashi-style marbling is more flowing with minimalist design. Remarkably,

This chapter is associated with Y. Sun, J. W. M. Bush, S. E. Spagnolie, and C. H. Rycroft, *The hydrodynamics of marbling art*, Milton van Dyke Award video winner of the Gallery of Fluid Motion (GFM) at the annual meeting of the American Physical Society's Division of Fluid Dynamics (2023). (doi:10.1103/APS.DFD.2023.GFM.V0002)

This chapter is also partly reformatted from an invited short paper about the aforementioned winning video for a special collection of the 2023 GFM winners, in preparation for Physics Review Fluids (2024).

both practices have remained relatively unchanged for centuries. In ebru-style marbling, marblers float paints on a viscosified water surface and use various thin tools (such as stylus, rake and comb) to craft classical or freestyle patterns. They then print the pattern onto mordanted paper specially treated with alum solution for better color adhesion [262]. In the modern era, many marblers use water mixed with carrageenan powder for the bath and diluted fluid acrylics for paint to marble on paper, fabric, ceramic, wood, glass, rock, and any possible surfaces imaginable.

Despite its rich historical and aesthetic lineage, the underlying fluid dynamics governing this art form has yet to be carefully studied. We celebrate the *marbelous* hydrodynamics of marbling art by focusing on two characteristic behaviors: The physics of spreading, which sets the initial color establishment based on the dominant balance between interfacial and inertial/viscous forces, and the physics of mixing, which allows us to draw delicate and complex patterns on the water surface since the Reynolds number is low. We showcase examples of classic marbling patterns and highlight the role of interfacial tension and small inertia in the making and science of marbling art. Through physical marbling experiments, we outline how a fluid–structure interaction simulation can be developed to model the art of marbling.

## 4.2 Marbling experiments

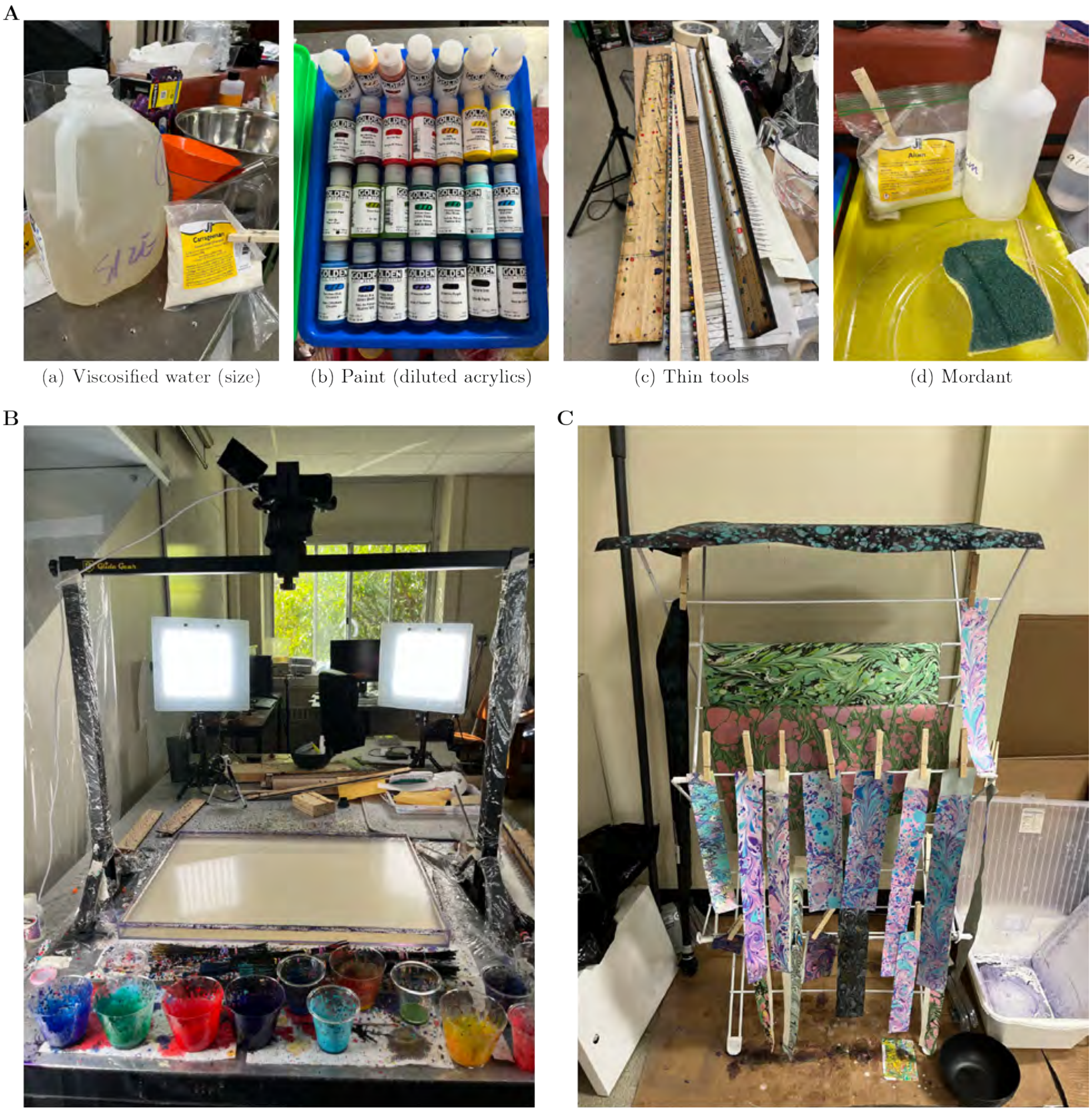
We conducted the marbling experiments at UW–Madison AMEP Lab[†]. We used a camera stand to film the top-down view from a fixed height, with four light sources to ensure uniform illumination especially for high-speed filming. Additionally, we used other cameras including iPhones to capture side

[†]The AMEP (Applied Math, Engineering, and Physics) Laboratory is housed in the mathematics department of the University of Wisconsin–Madison.

views, close-ups, and slow-motion footage; we also used micropipettes for the droplet spreading experiments[†]. For the GFM video and associated experiments, we mostly used an 18 in $\times$ 24 in $\times$ $1\frac{1}{3}$ in clear tray, but also used a smaller 11 in $\times$ 14 in $\times$ $1\frac{2}{5}$ in tray for initial tests and droplet spreading experiments. Fig. 4.1 shows the typical materials and setup of our marbling experiments.

Most marbling time is spent on preparation and cleanup. Each marbling session typically lasts for 2 h, with the preparation starting at least 24 h before the actual marbling. First, we need to prepare the marbling bath, which marblers refer to as the *size*, but we later refer to as *viscosified water* to avoid confusion. The viscosified water is made by mixing approximately 2 tablespoons of carrageenan powder in 1 gallon of water. After excessive mixing to ensure the carrageenan power is fully dissolved and the solution is homogenous, the viscosified water is left to rest for at least 24 h in the fridge to allow the carrageenan to fully hydrate and the solution to thicken. Before the marbling session, we need to take the viscosified water out of the fridge and let it rest at room temperature to ensure it is at the same temperature as the paint for best results. Second, we need to prepare the paint, which we use the Golden fluid acrylic[‡] diluted with distilled water in a 1 : 2 ratio. The paint can stay in room temperature for a couple of days, so we can prepare it in advance. However, before using it, we need to mix it well to ensure the paint is homogeneous and no paint pigments are settled at the bottom. Third, we mordant the paper with alum solution, which helps color to adhere onto the paper. Even though the mordant is important for the quality and longevity of printing marbled papers, we do not focus on the mordanting process in the current work. Finally, we

[†]Special thanks to Nadiya Mahomed and Professor Marta Gaglia for filming, lab, and experimental equipment support.
[‡]Golden fluid acrylic: `https://goldenartistcolors.com/products/golden-artist-acrylics/fluid`




A

(a) Viscosified water (size) (b) Paint (diluted acrylics) (c) Thin tools (d) Mordant

B

Marbling station

C

Drying racks

**Figure 4.1: Setup for marbling experiments.** Materials, marbling, and filming setup. **(A)** Marbling materials, including size, paint, thin tools, and mordant. **(B)** Marbling station, with camera and light setup for filming. **(C)** After rinsing, marbled papers are dried on drying racks.

prepare the tools and the marbling station, and then we are ready to marble. After printing the marbled pattern onto the mordanted paper, we rinse the paper with plain water to wash off excess viscosified water and paint, and then we dry the paper on drying racks before pressing it flat.

As mesmerizing as an art form, marbling actually presents a fluid–structure interaction (FSI) problem, where the viscosified water serves as the fluid, and the thin tools act as the solid. When we move these tools across the viscosified water surface, we are enacting an artistic rendition of FSI. Traditionally, marblers use tools like a stylus, rake, and comb to create intricate patterns. In addition to these traditional tools, we also laser-cut[†] wooden tools, allowing us to adjust the teeth distance on the rake or the comb. We also use eyedroppers and whisks made from brooms to apply colors. These eyedroppers are used to drop colors precisely onto the liquid surface, and the whisks are used to make the *stone* pattern and randomly drop colors onto the liquid surface.

We designed the marbling experiments after consulting with experienced marblers. Even after decades of practice, they still have remaining questions about the fluid dynamics of marbling that require careful consideration. Two questions stand out. Firstly, why do the paints float on the liquid surface even though they are denser? Secondly, why don't the colors mix even when they use a stylus to drag them along? We conducted various marbling experiments, ranging from learning the basic marbling techniques, to studying the role of interfacial tension and surfactants in the physics of spreading, to investigating the role of viscosity in the physics of mixing. The resulting marbled papers can be viewed at Appendix C.1.

[†]Special thanks to Dr. Giovanni Bordiga for laser cutting custom tools.

## 4.3 Theoretical formulation

In this section, we introduce two important concepts that underpin the physics of marbling: *interfacial tension* (physics of spreading) and *Stokes flow* (physics of mixing).

Interfacial tension is the material property of a fluid–fluid (liquid–gas or liquid–liquid) interface that reflects the energetic cost of creating that interface. It thus acts always to minimize the area of the interface. In our system, there are three surface tensions at play (paint–air, paint–bath, air–bath) (Fig. 4.2A). Hence we use the term interfacial tension to differentiate from surface tension. Surface tension is the material property of the liquid in contact with the gas phase (usually air), so at liquid–gas interface. For example, the surface tension at an air–water interface makes small raindrops spherical.

Stokes flow is the flow regime characterized by the assumption that there is little or no inertia, typically valid at low Reynolds numbers for systems for bacteria and other microorganisms. In our system, since we are dealing with viscosified water, we can approximate the Reynolds number with the width of the stylus (typically 0.2 cm) and the speed at which we move the stylus (typically 1 cm/s):

$$Re = \frac{UL}{\nu} = \frac{1\,\mathrm{cm/s} \times 0.2\,\mathrm{cm}}{1\,\mathrm{cm^2/s}} = 0.2, \tag{4.1}$$

where we take the viscosity based on the commercial carrageenan solution[†]. We model the viscosified bath with the incompressible Navier–Stokes equations. Since the Reynolds number is less than 1, we can

[†]Production, properties and uses of carrageenan : https://www.fao.org/3/x5822e/x5822e05.htm

assume that the inertia is small in the viscous bath because the inertial term in the governing equations vanishes:

$$Re \left( \underbrace{\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}}_{\text{inertia}} \right) = -\nabla p + \nabla^2 \boldsymbol{u} + \boldsymbol{f}, \qquad \nabla \cdot \boldsymbol{u} = 0. \tag{4.2}$$

A nice property of Stokes flow is that the fluid motion only responds to instantaneous boundary conditions. This is where the FSI comes in: Given the boundary conditions, which can be prescribed by the solid (*e.g.* thin tools), we can solve for the corresponding flow profile. The Stokes flow assumption can serve as a good foundation for future work on analytical solving for the flow profile given the movement of the thin tools, which we will discuss in Section 4.6.

## 4.4 Results

In this section, we present the results of our marbling experiments, focusing on the physics of spreading and mixing. We first discuss the physics of spreading, examining the role of interfacial tensions and surfactants in the spreading process. Then, we compare the flow patterns at high and low viscosity baths to uncover the physics of mixing. Insights from these experiments provide valuable information for developing more physically accurate numerical simulations of marbling, to be outlined in Section 4.5.

Before delving into the physics of spreading, let us first examine why the denser paint droplet floats on the viscosified water surface rather than sinks to the bottom of the bath. While the paint is slightly denser than the bath, the balance of interfacial tensions is sufficient to support its weight (Fig. 4.2A). Moreover, it is energetically favorable for the paint to spread on the surface rather than sink to the bottom. The paint spreads due to the imbalance in interfacial tensions at its edge. This spreading force is resisted in turn by inertial and then viscous forces, until the paint achieves a static equilibrium in which all interfacial forces are balanced (Fig. 4.2B–C).

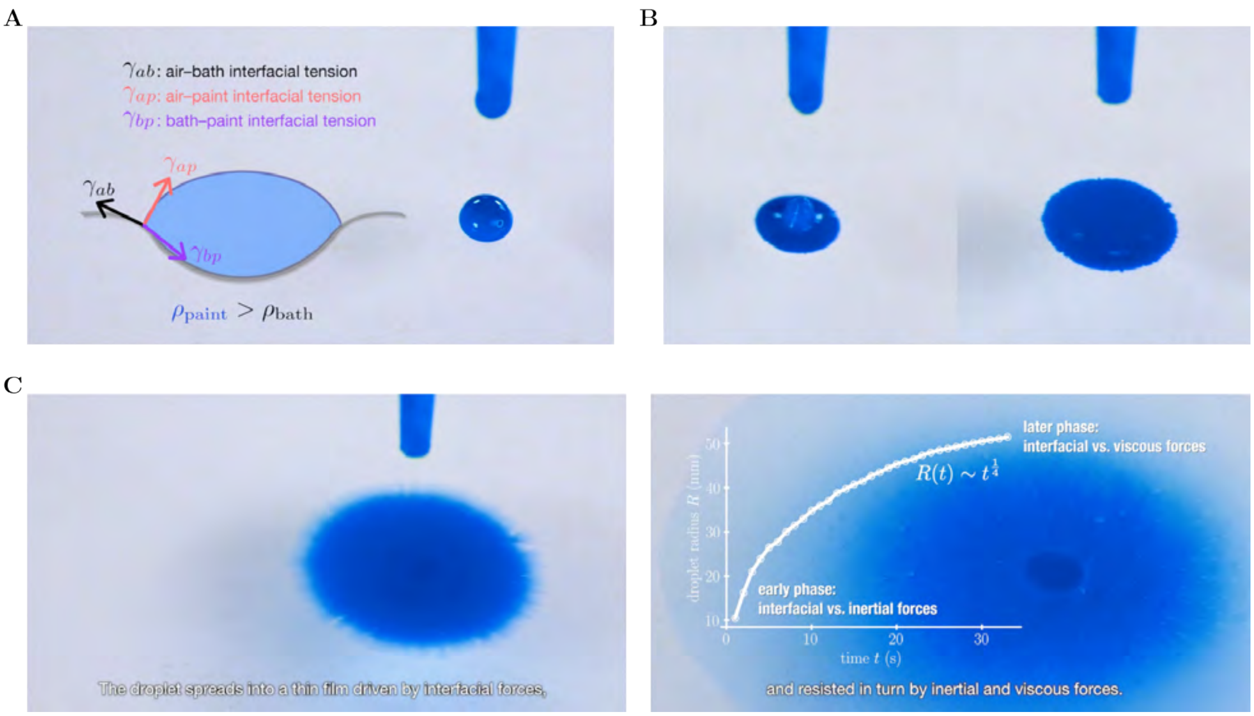



**Figure 4.2: Physics of spreading.** High-speed video of dropping one droplet onto the viscosified bath. **(A)** The balance of interfacial tensions supports the weight of the droplet. **(B)** Snapshots of the droplet spreading on the bath surface. **(C)** The droplet spreads until the interfacial forces are balanced at its edge, first by inertial then viscous forces.

We can modify the surface tension of the paint by adding surfactants, which lower surface tension and promote spreading. This explains why we observe the paint spreading more quickly when surfactants are added (Fig. 4.3A). Initially, the blue paint has a higher surface tension than the red, causing it to spread less. However, after adding surfactants[†], the blue paint spreads more effectively compared to previous

[†]For modern ebru-style marbling, marblers often use Photo-Flo or synthetic gall as surfactants, instead of natural ox gall.

experiments. Since different colors can have varying surface tensions, marbling becomes a game of global balance of surface tensions. Therefore, when we add new colors (Fig. 4.3B) either using eyedroppers or whisking paints, the resulting Marangoni stresses cause rearrangement of the surface pattern, which may result in expansion or compression of paints.

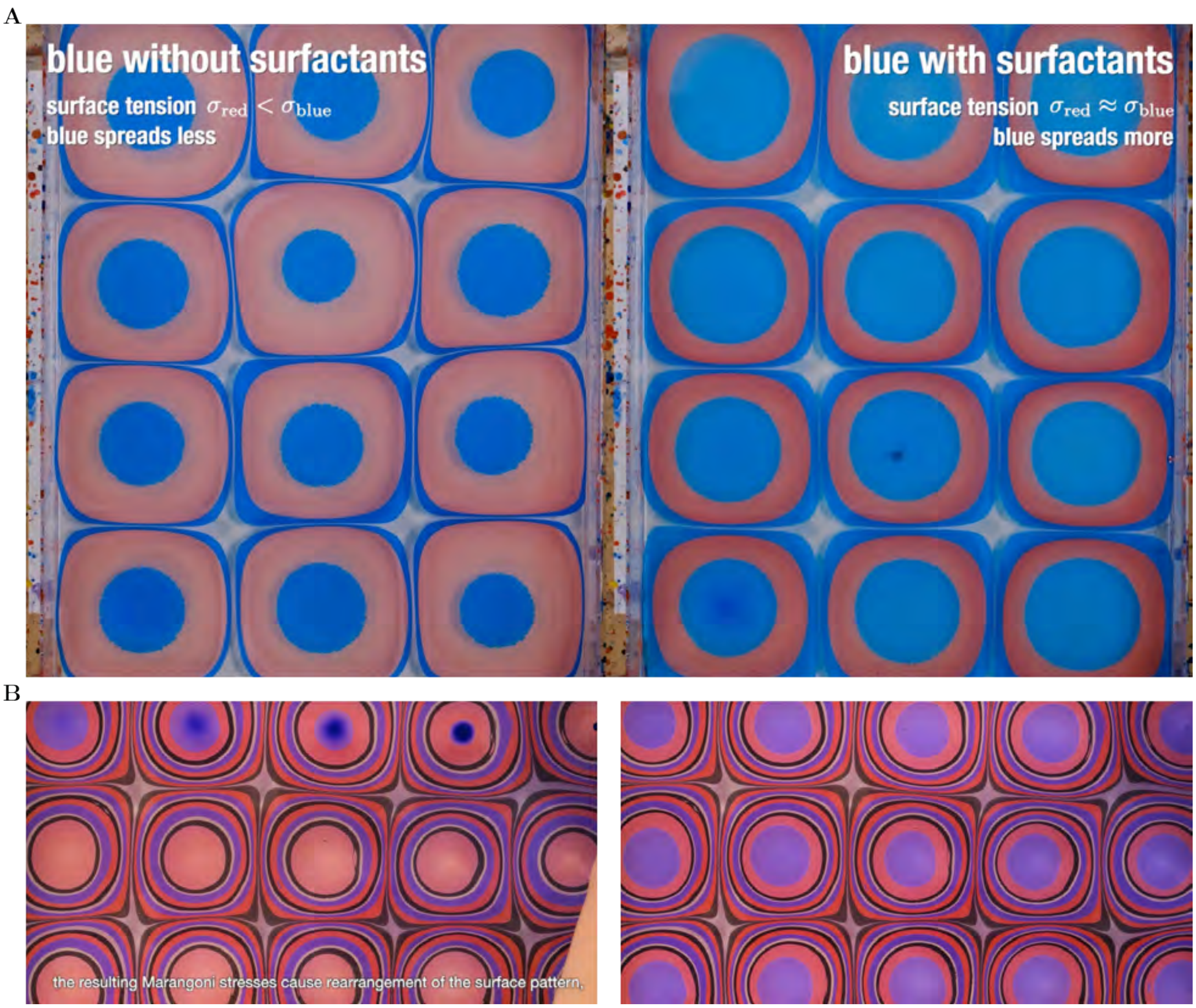



**Figure 4.3: Role of surfactants in spreading. (A)** Effects of surfactants in blue paints spreading. **(B)** Effects of different surface tensions in rearrangement of the surface pattern.

After we have set the initial color pattern, the next step in marbling is to use thin tools (solids) to create the patterns. The main why these patterns are stable is that the bath is very viscous. As we explained in Section 4.3, the Reynolds number is less than 1, so the inertial term in the governing equations vanishes. Because inertia is small in the viscous bath, fluid motion only responds to instantaneous boundary conditions. Therefore, at high viscosity, we have more control over the patterns. However, at lower viscosity, inertial effects take over, where we observe the wake of instability happening behind the stylus (Fig. 4.4). In addition, because the paint is spread into a thin layer on the bath, there is not enough contact area between different colors to allow for significant mixing (Fig. 4.5). Diffusion acts on a longer timescale than the time takes to marble, so detailed patterns are stable.

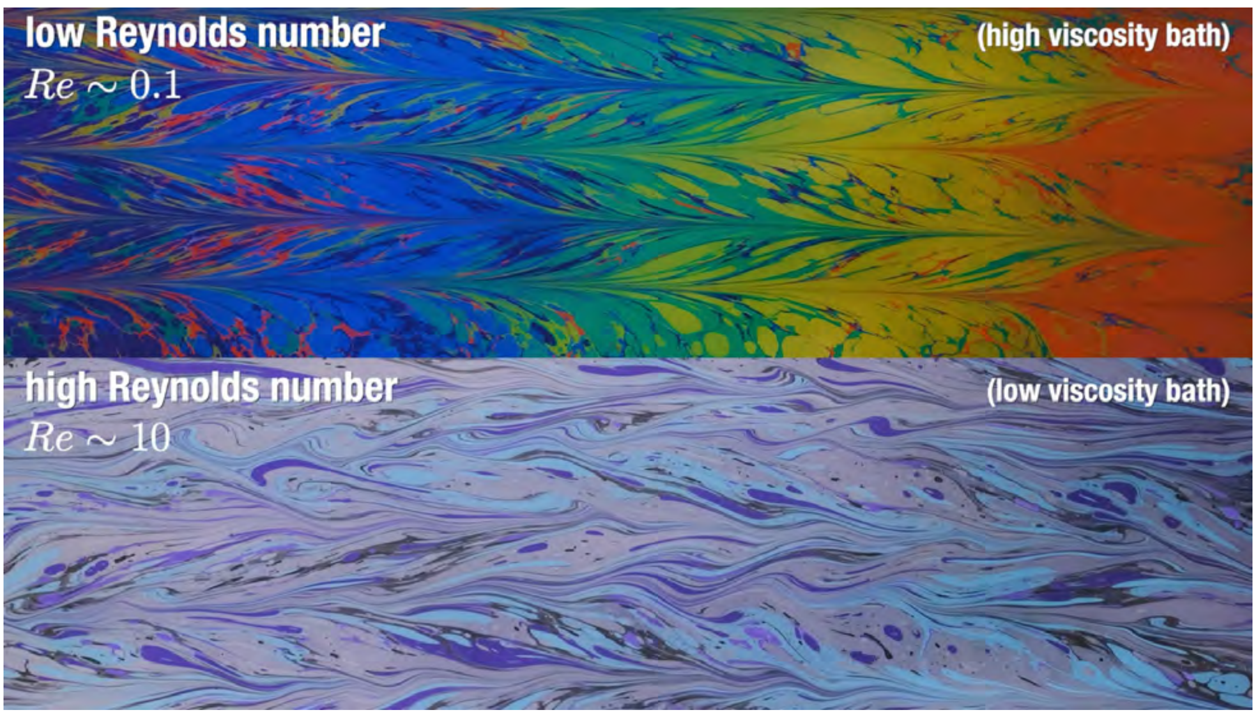



**Figure 4.4: Physics of (not) mixing.** Comparison of flow patterns at high and low viscosity bath.

**Figure 4.5: Example of a stable swirling pattern.** Surface diffusion acts on a longer timescale than the time takes to marble, so detailed patterns are stable.

## 4.5 Outlook of numerical implementation

Now that we have witnessed marbling in action and learned about some of the fluid dynamics behind it, it is probably time to discuss why we have chosen an alternative approach—namely, to focus not on simulation, but on the art and physics of marbling. Our first encounter with marbling was in a homework assignment back in Spring 2019 for Harvard Applied Math 225 "Advanced Numerical Methods II", where the question was to simulate paper marbling. Despite successfully completing the assignment, the resulting simulation fell short of capturing the essence of marbling as seen in online videos: The color interfaces were blurry, and the patterns appeared less stable but more wavy. Nonetheless, the mesmerizing marbling videos online piqued our interest to learn the art ourselves. It is through this hands-on experience that we can become closer to understanding the physics of marbling art.

And indeed, we did. In addition to making art in the lab and producing hundreds of marbled papers, we are getting closer to developing a more true-to-physics marbling simulation with insights learned in the marbling experiments. We outline a numerical implementation of `marbling-fluids` for future work on the multiphysics simulation of FSI in marbling. It is worth noting that there are existing marbling simulations capable of digitally creating stunning marbling patterns. Researchers in computer graphics and interactive art have developed programs for digital marbling in two dimensions (2D). One computational approach models color as a passive scalar field atop a background fluid flow prescribed with virtual stroke movements [263–265]. Another approach employs mathematical homeomorphic bijection to approximate the patterns [266, 267]. Final renders from both methods closely resemble actual handmade marbled patterns. However, these approaches simplify the marbling process and fail to dynamically capture the interfacial tension while maintaining a sharp interface between different color droplets [268].

In `marbling-fluids`, we propose a similar approach to modeling the marbling process as a 2D FSI problem. As the marbling paints spread thinly across the viscosified water surface, we model the background fluid flow using the incompressible Navier–Stokes equations:

$$\begin{aligned} \rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right) &= -\nabla p + \mu\nabla^2\boldsymbol{u}, \\ \nabla\cdot\boldsymbol{u} &= 0, \end{aligned} \tag{4.3}$$

where $\rho$ is the fluid density, $\mu$ is the fluid dynamic viscosity, and $\boldsymbol{u}(\boldsymbol{x}, t)$ and $p(\boldsymbol{x}, t)$ respectively represent the fluid velocity and pressure fields. The choice of $\mu$ needs to be carefully calibrated to match the viscosity of the viscosified water, *i.e.* it needs to be large to model the high viscosity in the marbling bath. We couple Eq. (4.3) with the advection of a three-component vector field $\boldsymbol{c}(\boldsymbol{x}, t) = (R(\boldsymbol{x}, t), G(\boldsymbol{x}, t), B(\boldsymbol{x}, t))$, representing the red, green, and blue color channels. This color vector field is subject to the hyperbolic conservation law:

$$\frac{\partial \boldsymbol{c}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\, \boldsymbol{c} = \mathbf{0}. \tag{4.4}$$

After solving for Eq. (4.3), we use the updated velocity field to advect the color field. For the time derivative, we opt for the explicit Euler method, while for the spatial derivative, we use the upwinding essentially non-oscillatory (ENO) method [269], which helps to maintain sharp color discontinuities.

However, simply simulating Eqs. (4.3) and (4.4) will not suffice to capture the interfacial phenomenon of marbling: We are likely to observe blurring of color boundaries. We propose to incorporate the reference map technique (RMT) [96] to track the color pattern in `marbling-fluids`. Rather than directly advecting the color field, we opt to advect a reference map field $\boldsymbol{\xi}(\boldsymbol{x}, t)$, denoted as a two-component vector field. $\boldsymbol{\xi}(\boldsymbol{x}, t)$ maps the current deformed color field back to its initial reference state—akin to the approach used for simulating solids in the Eulerian framework in Chapter 3. Since the reference color pattern is constant, the reference map field satisfies the advection equation:

$$\frac{\partial \boldsymbol{\xi}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\, \boldsymbol{\xi} = \mathbf{0}. \tag{4.5}$$

Suppose we initialize the reference map field to $\boldsymbol{\xi}(\boldsymbol{x}, 0)$ at time $t = 0$, $\boldsymbol{\xi}$ tracks the deformation of the color field, indicating the reference position of the color at $t = 0$ that is currently at $\boldsymbol{x}$ at $t = T$. Moreover, we propose to incorporate a surface tension term into the boundary condition for the reference map field. This term will effectively to capture the dynamic interfacial tension between different color droplets. Once $\boldsymbol{\xi}(\boldsymbol{x}, T)$ is solved via Eq. (4.5), we can find the color at $t = T$ as

$$\boldsymbol{c}(\boldsymbol{x}, T) = \boldsymbol{c}\left(\boldsymbol{\xi}(\boldsymbol{x}, T), 0\right). \tag{4.6}$$

Here we outline a numerical implementation to simulate marbling as a coupled FSI problem using Eqs. (4.3) and (4.5). Additional features to be included in `marbling-fluids` are functionalities to prescribe boundary conditions for the fluid velocity field as marbling strokes, as well as to develop a method for chaining reference map fields. These features will enable us to model extreme distortion of the color pattern, a phenomenon quite common in marbling, where thin tools repeatedly move across the bath.

## 4.6 Discussion

In this chapter, we have explored the *marbelous* fluid dynamics of marbling art. We have shown that the physics of spreading and mixing are the two key ingredients that make marbling art possible. The physics of spreading sets the initial color establishment based on the balance between interfacial and inertial/viscous forces, while the physics of mixing allows us to draw on the water surface since the Reynolds number is low. We have also demonstrated that the interfacial tension and small inertia play a crucial role in the making and science of marbling art through a gallery of marbling patterns.

No two marbled papers are the same. There is no way to apply color the same way or move the thin tools in exactly the same manner. Myriad color choices, pattern designs, stroke movements, and environmental conditions all contribute to the uniqueness of marbling art. This uniqueness makes marbling mesmerizing but also frustrating for those who seek perfection. Rather than strive for perfection, we learn to embrace the randomness of marbling and let these *happy accidents* guide us in the marbling process.

We have also taken a different route in studying fluid–structure interaction and focused on the physics of an art form. For future work, we aim to integrate our real-life marbling experiences into the simulation. Firstly, we plan to enhance the accuracy by incorporating the correct physical parameters, such as the viscosity of the bath and the interfacial tensions. Secondly, we intend to introduce functionalities that allow us to mimic marbling strokes as prescribed boundary conditions. Our goal is to develop a simulation that enables us to observe how paints spread and contract on the liquid bath while accurately modeling interfacial tension and the global balance of Marangoni stresses. Furthermore, we aim to maintain a sharp interface between colors throughout the simulation with the reference map technique.

In addition to simulation work, we aim to conduct a more systematic study of paint spreading on the viscosified water surface and compare its spreading behavior with established literature, such as oil spills on the ocean surface [270, 271]. Additionally, we aim to develop an analytical model to solve for the flow pattern based on the marbling strokes (instantaneous boundary conditions). With those, our final goal is to develop an inverse design tool that can output strokes needed for creating a targeted marbling pattern.